

11.2 Angle Theorems for Triangles

Common Core Standards

8.G.5

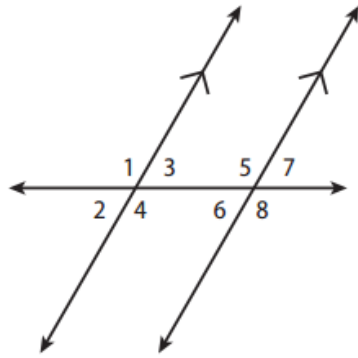
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles

8.EE.7

Solve linear equations in one variable

8.EE.7b

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms



**Get out a half sheet of paper
and answer these questions for
a QUIZ grade.**

- 1.** Name both pairs of alternate interior angles.
- 2.** Name all pairs of corresponding angles.
- 3.** Name the relationship between $\angle 2$ and $\angle 7$.

11.2 Angle Theorems for Triangles

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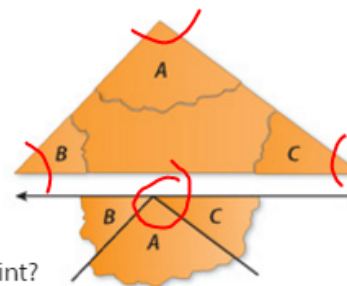
EXPLORE ACTIVITY 1

COMMON CORE 8.G.5

Sum of the Angle Measures in a Triangle

There is a special relationship between the measures of the interior angles of a triangle.

- A** Draw a triangle and cut it out. Label the angles A , B , and C .
- B** Tear off each "corner" of the triangle. Each corner includes the vertex of one angle of the triangle.
- C** Arrange the vertices of the triangle around a point so that none of your corners overlap and there are no gaps between them.
- D** What do you notice about how the angles fit together around a point?



E What is the measure of a straight angle? 180°

F Describe the relationship among the measures of the angles of $\triangle ABC$.

measures to equal 180°

The Triangle Sum Theorem states that for $\triangle ABC$, $m\angle A + m\angle B + m\angle C = 180^\circ$.

1/2

Follow the steps to informally prove the Triangle Sum Theorem. You should draw each step on your own paper. The figures below are provided for you to check your work.

A Draw a triangle and label the angles as $\angle 1$, $\angle 2$, and $\angle 3$ as shown.

B Draw line a through the base of the triangle.

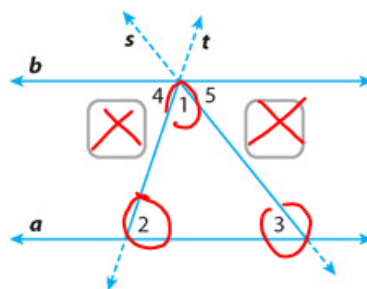
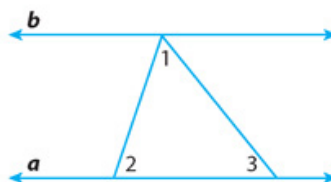
C The Parallel Postulate states that through a point not on a line ℓ , there is exactly one line parallel to line ℓ . Draw line b parallel to line a , through the vertex opposite the base of the triangle.

D Extend each of the non-base sides of the triangle to form transversal s and transversal t . Transversals s and t intersect parallel lines a and b .

E Label the angles formed by line b and the transversals as $\angle 4$ and $\angle 5$.

F Because $\angle 4$ and $\angle 2$ are alternate interior angles, they are Congruent.

Label $\angle 4$ with the number of the angle to which it is congruent.



G Because $\angle 5$ and $\angle 3$ are alternate interior angles, they are Congruent.

Label $\angle 5$ with the number of the angle to which it is congruent.

H The three angles that lie along line b at the vertex of the triangle are $\angle 1$, $\angle 4$, and $\angle 5$. Notice that these three angles lie along a line.

$$\text{So, } m\angle 1 + m\angle 4 + m\angle 5 = 180^\circ$$

Because angles 2 and 4 are congruent and angles 3 and 5 are congruent, you can substitute $m\angle 2$ for $m\angle 4$ and $m\angle 3$ for $m\angle 5$ in the equation above.

$$\text{So, } m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$$

This shows that the sum of the angle measures in a triangle is always 180° .

$$\angle 2 \cong \angle 4$$

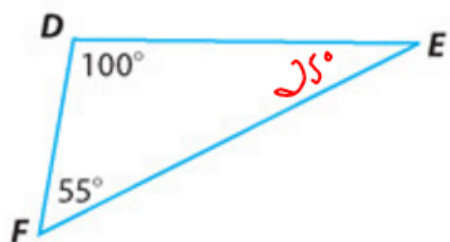
$$\angle 5 \cong \angle 3$$

Find the missing angle measure.

STEP 1

Write the Triangle Sum Theorem for this triangle.

$$\text{✂ } m\angle D + m\angle E + m\angle F = 180^\circ$$



STEP 2

Substitute the given angle measures.

$$100 + m\angle E + 55 = 180$$

STEP 3

Solve the equation for $m\angle E$.

$$\begin{array}{r} 100 + \angle E = 180 \\ -100 \qquad -100 \\ \hline m\angle E = 25^\circ \end{array}$$

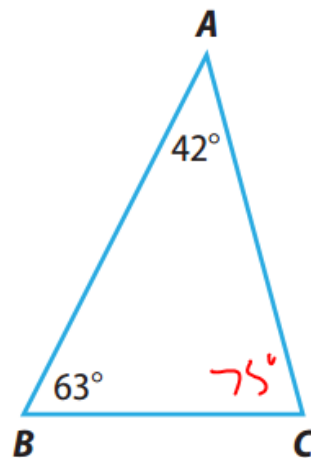
Find the missing angle measure.

$$m\angle A + m\angle B + m\angle C = 180$$

$$42 + 63 + \angle C = 180$$

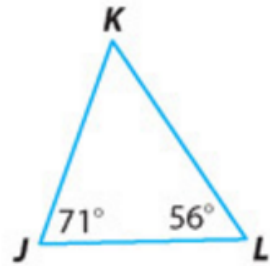
$$105 + \angle C = 180$$
$$-105 \quad -105$$

$$m\angle C = 75^\circ$$



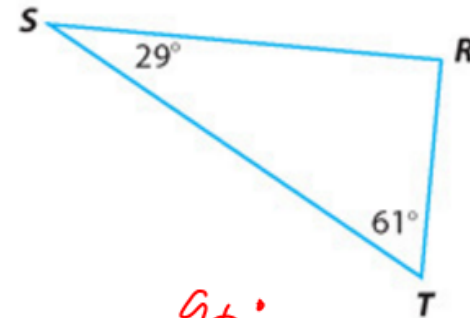
Find the missing angle measure.

4.



$$m\angle K = \underline{53^\circ}$$

5.



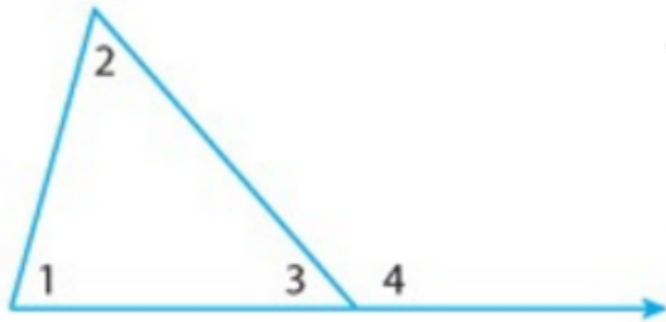
$$m\angle R = \underline{90^\circ}$$

Interior Angle - inside a triangle formed by 2 sides of the triangle

Exterior Angle - formed by 1 side of the triangle and the extension of an adjacent side



Remote Interior Angle - an interior angle that is not adjacent to the exterior angle



- $\angle 1$, $\angle 2$, and $\angle 3$ are interior angles.
- $\angle 4$ is an exterior angle.
- $\angle 1$ and $\angle 2$ are remote interior angles to $\angle 4$.

There is a special relationship between the measure of an exterior angle and the measures of its remote interior angles.



A Extend the base of the triangle and label the exterior angle as $\angle 4$.

B The Triangle Sum Theorem states:

$$m\angle 1 + m\angle 2 + m\angle 3 = \underline{180^\circ}.$$

C $\angle 3$ and $\angle 4$ form a straight angle,

$$\text{so } m\angle 3 + m\angle 4 = \underline{180^\circ}.$$

D Use the equations in **B** and **C** to complete the following equation:

$$m\angle 1 + m\angle 2 + \underline{m\angle 3} = \underline{m\angle 3} + m\angle 4$$

E Use properties of equality to simplify the equation in **D**:

$$\underline{m\angle 1 + m\angle 2 = m\angle 4}$$

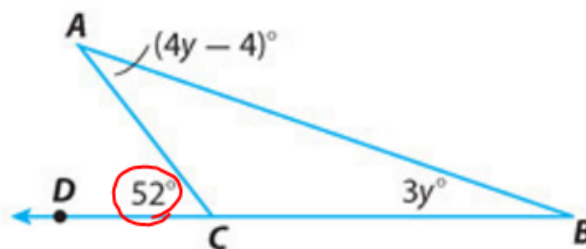
The Exterior Angle Theorem states that the measure of an exterior angle is equal to the sum of its remote interior angles.

Find $m\angle A$ and $m\angle B$.

STEP 1

Write the Exterior Angle Theorem as it applies to this triangle.

$$m\angle A + m\angle B = m\angle ACD$$



STEP 2

Substitute the given angle measures.

STEP 3

Solve the equation for y .

STEP 4

Use the value of y to find $m\angle A$ and $m\angle B$.

$$\angle A = 4(8) - 4 = 32 - 4 = 28^\circ$$

$$\angle B = 3(8) = 24^\circ$$

$$4y - 4 + 3y = 52$$

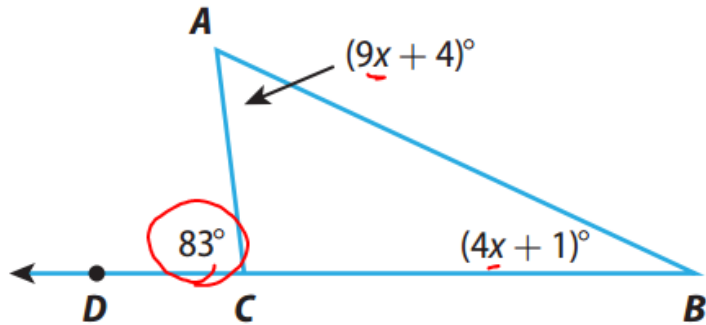
$$7y - 4 = 52$$

$$+4 \quad +4$$

$$\frac{7y}{7} = \frac{56}{7}$$

$$y = 8$$

Find $m\angle A$ and $m\angle B$.



$$m\angle A = 9(6) + 4 = 58^\circ$$

$$m\angle B = 4(6) + 1 = 25^\circ$$

$$m\angle ACD = m\angle A + m\angle B$$

$$83 = 9x + 4 + 4x + 1$$

$$\begin{array}{r} 83 = 13x + 5 \\ \underline{-5} \end{array}$$

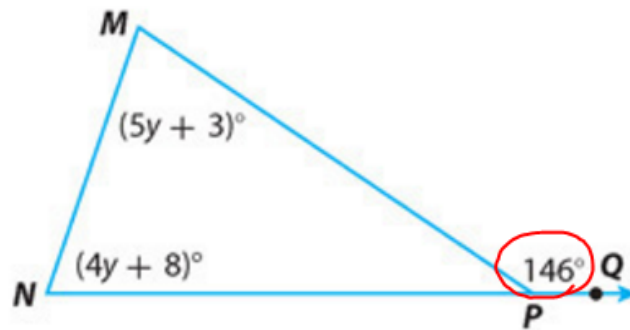
$$\frac{78}{13} = \frac{13x}{13}$$

$$x = 6$$

8. Find $m\angle M$ and $m\angle N$.

$$m\angle M = \underline{78^\circ}$$

$$m\angle N = \underline{68^\circ}$$



$$m\angle M = 5(15) + 3 = 78^\circ$$

$$m\angle N = 4(15) + 8 = 68^\circ$$

$$m\angle MPQ = m\angle M + m\angle N$$

$$146 = 5y + 3 + 4y + 8$$

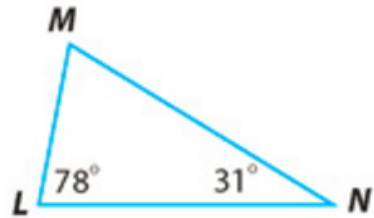
$$\begin{array}{r} 146 = 9y + 11 \\ -11 \\ \hline \end{array}$$

$$\frac{135}{9} = \frac{94}{9}$$

$$y = 15$$

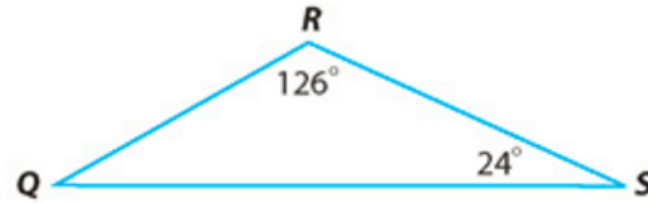
Find each missing angle measure. ([Explore Activity 1 and Example 1](#))

1.



$$m\angle M = \underline{\hspace{2cm}}$$

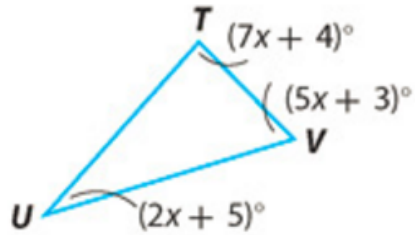
2.



$$m\angle Q = \underline{\hspace{2cm}}$$

Use the Triangle Sum Theorem to find the measure of each angle in degrees. (Explore Activity 2 and Example 1)

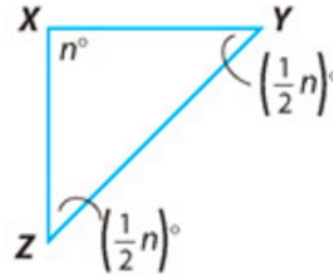
3.



$$m\angle T = \underline{\hspace{2cm}}, m\angle U = \underline{\hspace{2cm}},$$

$$m\angle V = \underline{\hspace{2cm}}$$

4.



$$m\angle X = \underline{\hspace{2cm}}, m\angle Y = \underline{\hspace{2cm}},$$

$$m\angle Z = \underline{\hspace{2cm}}$$

$$7x + 4 + 5x + 3 + 2x + 5 = 180$$

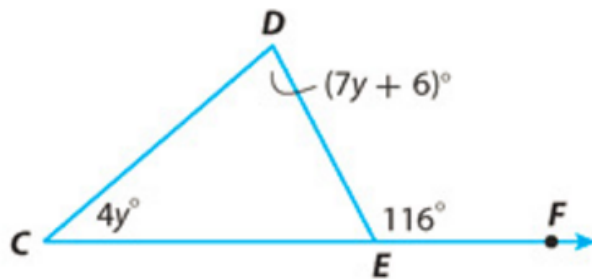
$$14x + 12 = 180$$

$$\begin{array}{r} -12 \\ -12 \end{array}$$

$$\begin{array}{r} 14x = 168 \\ \hline 14 \quad \hline 14 \\ \hline x = 12 \end{array}$$

Use the Exterior Angle Theorem to find the measure of each angle in degrees. ([Explore Activity 3](#) and [Example 2](#))

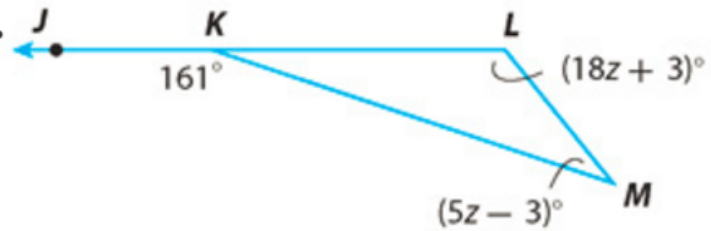
5.



$$m\angle C = \underline{\hspace{2cm}}, m\angle D = \underline{\hspace{2cm}},$$

$$m\angle DEC = \underline{\hspace{2cm}}$$

6.



$$m\angle L = \underline{\hspace{2cm}}, m\angle M = \underline{\hspace{2cm}},$$

$$m\angle LKM = \underline{\hspace{2cm}}$$

H/W

p 358 - 359

1 - 14

★ Finish
transversal
HW