

Bellringer

1. Nico earns \$12.50 per hour as a math tutor. Show that the relationship between the amount he earns and the number of hours he tutors is a proportional relationship. Then write the equation for the relationship.
2. The table below shows a proportional relationship. Write an equation that describes the relationship.

Acres	5	8	15
Bushels of Wheat	140	224	420

Bellringer

Open: *student.masteryconnect.com*

Login: use code _____

You have 10 minutes

- * Once you finish, get your notes and textbook out
- * Don't forget the assignment that is due Friday

3.2 Rate of Change and Slope

8.F.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values

Vocabulary

- Rate of Change - ratio of the amount of change in the dependent variable (output), to the change in the independent variable (input)
- Slope - ratio of the change in y-values (rise) for a segment of a graph to the corresponding change in the x-values (run)
 - ↳ represented by a line on a graph

Formulas

slope
↓

① $m = \frac{\text{Rise}}{\text{Run}} \rightarrow \text{graph}$

② $m = \frac{\Delta y}{\Delta x} = \frac{\text{Change in } y}{\text{Change in } x} \rightarrow \text{table}$

③ $m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \text{ordered pairs } (x, y)$

EXAMPLE 1COMMON
CORE

8.F.4

Eve keeps a record of the number of lawns she has mowed and the money she has earned. Tell whether the rates of change are constant or variable.

	Day 1	Day 2	Day 3	Day 4
X Number of lawns	1	3	6	8
Y Amount earned (\$)	15	45	90	120

Handwritten annotations: Red arrows show changes between days: +2 (1 to 3), +3 (3 to 6), +2 (6 to 8) for lawns; +30 (15 to 45), +45 (45 to 90), +30 (90 to 120) for amount earned. These are circled in green. A red fraction $\frac{\Delta Y}{\Delta X}$ is written to the right.

STEP 1

Identify input/output variables
 lawns \leftarrow \rightarrow \$

Constant R.o.C.

STEP 2

Find Rate of Change

$$\frac{30}{2} = \boxed{\frac{15}{1}} \quad \frac{45}{3} = \boxed{\frac{15}{1}} \quad \frac{30}{2} = \boxed{\frac{15}{1}}$$

$$y = 15x$$

ADDITIONAL EXAMPLE 1

Hector keeps a record of the total number of clients he has and the amount he earns as a personal trainer. Tell whether the rates of change are constant or variable.

	Day 1	Day 2	Day 3	Day 4
Number of clients x	1	3	4	7
Amount earned (\$) y	45	135	180	315

Handwritten annotations in green and blue:

- Between Day 1 and Day 2: $+2$ (clients), $+90$ (amount)
- Between Day 2 and Day 3: $+1$ (clients), $+45$ (amount)
- Between Day 3 and Day 4: $+3$ (clients), $+135$ (amount)

① Find Rates of Change

② $\left[\frac{\Delta y}{\Delta x} \right]$ to make ratios

$$\frac{90}{2} = \left[\frac{45}{1} \right]$$

$$\left[\frac{45}{1} \right]$$

Constant
R.O.C.

$$\frac{135}{3} = \left[\frac{45}{1} \right]$$

$y = 45x$
\$45/client

YOUR TURN

1. The table shows the approximate height of a football after it is kicked. Tell whether the rates of change are constant or variable.

Find the rates of change:

$$\frac{18}{.5} \quad \frac{13}{1} \quad \frac{-5}{.5}$$

The rates of change are **constant / variable.**

<i>x</i> Time (s)	<i>y</i> Height (ft)
0	0
0.5	18
1.5	31
2	26

Handwritten annotations:
 - Between 0 and 0.5: $+5$ (left), $+18$ (right)
 - Between 0.5 and 1.5: $+1$ (left), $+13$ (right)
 - Between 1.5 and 2: $+5$ (left), -5 (right)

EXPLORE ACTIVITY



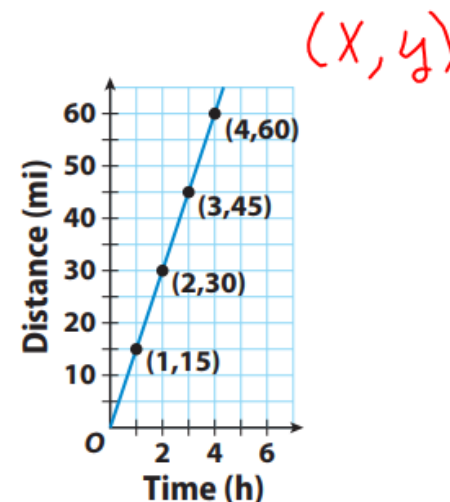
COMMON CORE 8.F.4

Using Graphs to Find Rates of Change

You can also use a graph to find rates of change.

The graph shows the distance Nathan bicycled over time.
What is Nathan's rate of change?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



- A** Find the rate of change from 1 hour to 2 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{30 - 15}{2 - 1} = \frac{15}{1} = 15 \text{ miles per hour}$$

- B** Find the rate of change from 1 hour to 4 hours.

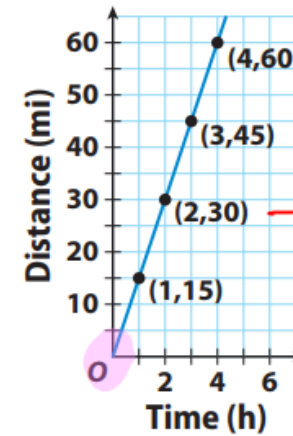
$$\frac{\text{change in distance}}{\text{change in time}} = \frac{60 - 15}{4 - 1} = \frac{45}{3} = 15 \text{ miles per hour}$$

- C** Find the rate of change from 2 hours to 4 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{60 - \boxed{30}}{4 - \boxed{2}} = \frac{\boxed{30}}{\boxed{2}} = \boxed{15} \text{ miles per hour}$$

- D** Recall that the graph of a proportional relationship is a line through the origin. Explain whether the relationship between Nathan's time and distance is a proportional relationship.

yes

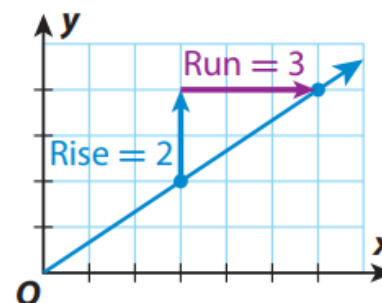


$$K = \frac{15}{1} \quad \boxed{\frac{15}{1}} \quad \frac{30}{2} \quad \frac{45}{3} \quad \frac{60}{4}$$

$$\boxed{\frac{15}{1}} \quad \boxed{\frac{15}{1}} \quad \boxed{\frac{15}{1}}$$

Calculating Slope m

When the rate of change of a relationship is constant, any segment of its graph has the same steepness. The constant rate of change is called the *slope* of the line.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\text{Rise}}{\text{Run}}$$



$$\frac{\text{Rise } \uparrow (+) \downarrow (-)}{\text{Run } \rightarrow (+) \leftarrow (-)}$$

EXAMPLE 2COMMON
CORE

8.F.4

 (x, y) Find m the slope of the line.**STEP 1**

Find 2 points that cross on a corner

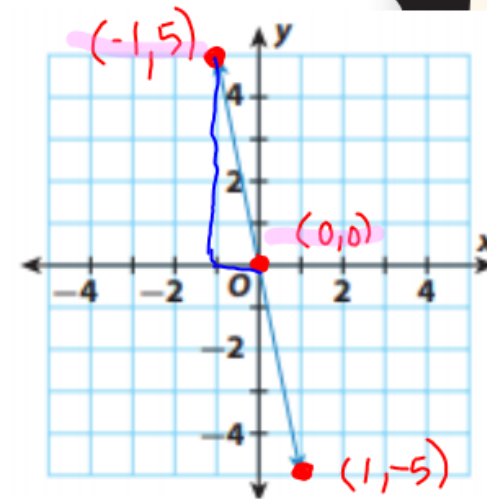
STEP 2

Solve for slope

$$\textcircled{1} \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{-1 - 0}$$

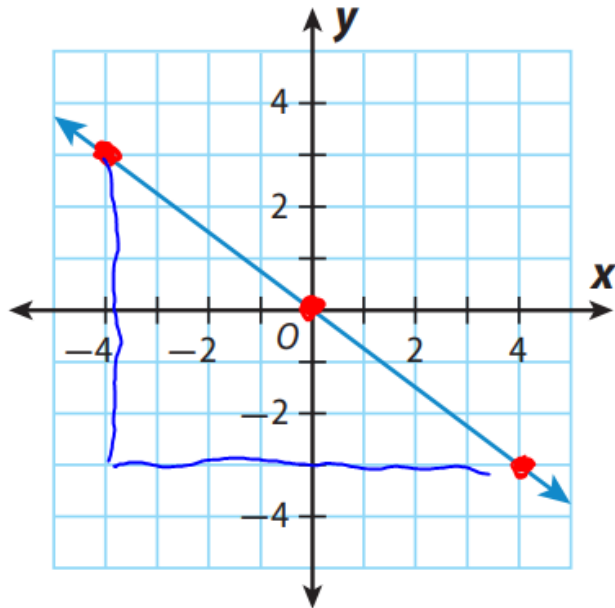
$$m = -\frac{5}{1}$$

$$\textcircled{2} \frac{\text{Rise}}{\text{Run}} = \frac{-5}{1}$$



ADDITIONAL EXAMPLE 2

Find the slope of the line.



- Find where line cross corners
- Make points
- Use $\frac{\text{Rise}}{\text{Run}}$ to find slope

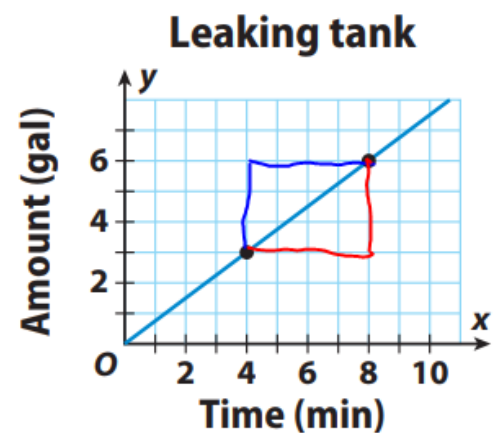
$$\frac{-6}{8} \div 2 = m = \frac{-3}{4}$$

YOUR TURN

4. The graph shows the rate at which water is leaking from a tank. The slope of the line gives the leaking rate in gallons per minute. Find the slope of the line.

$$\text{Rise} = \underline{3} \qquad \text{Run} = \underline{4}$$

$$\text{Slope} = \underline{\frac{3}{4}}$$



$$\left[\frac{-3}{-4} \right] = \frac{3}{4}$$

Guided Practice

Tell whether the rates of change are constant or variable. (Example 1)

1. building measurements _____

Feet	3	12	27	75
Yards	1	4	9	25

Handwritten notes for problem 1:
 Above the table, "Constant" is written in red. Arched red lines connect the values in the "Feet" row: 3 to 12 (+9), 12 to 27 (+15), and 27 to 75 (+48).
 Below the table, red arrows point from the "Feet" row to the "Yards" row: 3 to 1 (+3), 12 to 4 (+5), and 27 to 9 (+16).

2. computers sold _____

Week	2	4	9	20
Number Sold	6	12	25	60

3. distance an object falls _____

Distance (ft)	16	64	144	256
Time (s)	1	2	3	4

4. cost of sweaters _____

Number	2	4	7	9
Cost (\$)	38	76	133	171

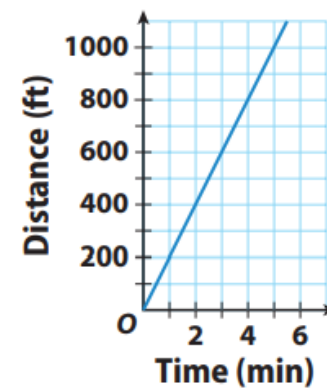
Handwritten calculations for problem 3:
 $\frac{9}{3} = \frac{3}{1}$ $\frac{15}{5} = \frac{3}{1}$ $\frac{48}{16} = \frac{3}{1}$

Erica walks to her friend Philip's house. The graph shows Erica's distance from home over time. (Explore Activity)

5. Find the rate of change from 1 minute to 2 minutes.

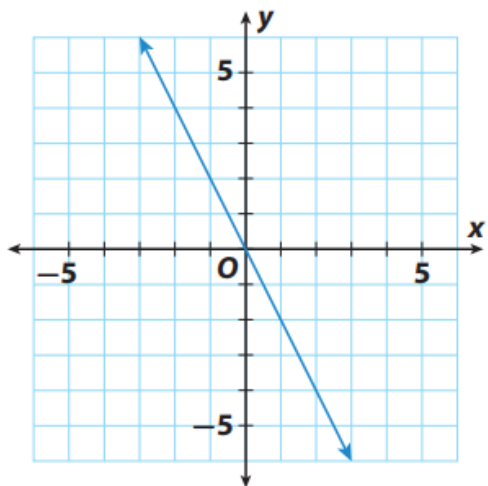
$$\frac{\text{change in distance}}{\text{change in time}} = \frac{400 - \boxed{}}{2 - \boxed{}} = \frac{\boxed{}}{\boxed{}} = \boxed{} \text{ ft per min}$$

6. Find the rate of change from 1 minute to 4 minutes. _____



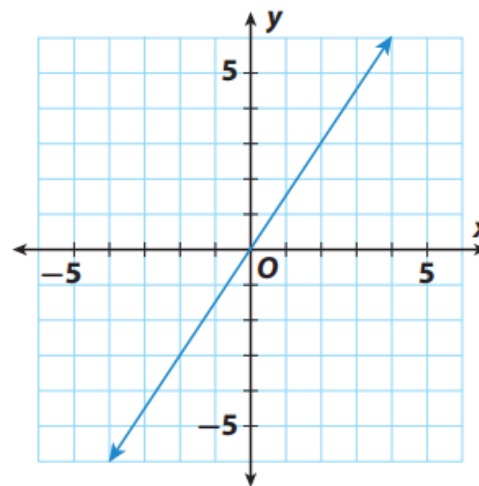
Find the slope of each line. (Example 2)

7.



slope = _____

8.



slope = _____

p80

1-8