$y = mx + b$

Bellringer

$14 = 12(1) + b$

$14 = 12 + b$

$-12 \quad -12$

$2 = b$

The table shows Eli's distance from home as he rides his bike at a steady rate after meeting a friend.

x	**Time (h)**	1	2	3	4	5
y	**Distance (mi)**	14	26	38	50	62

0 +1 +1 +1 +1 +1

+12 +12 +12 +12

1. Graph the data.
2. Find the slope and y-intercept. $m = \frac{\Delta y}{\Delta x} = \frac{12}{1}$ $b = 2$
3. How fast does Eli ride his bike, in miles per hour? 12 mph
4. Write an equation in $y = mx + b$ form that represents the miles, y, that Eli goes in x hours. $y = 12x + 2$

5.3 Linear Relationships and Bivariate Data

8.SP.1
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points on the line.

8.SP.3
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept

Finding the Equation of a Linear Relationship

* Use points on a graph of a relationship to write an equation for the relationship

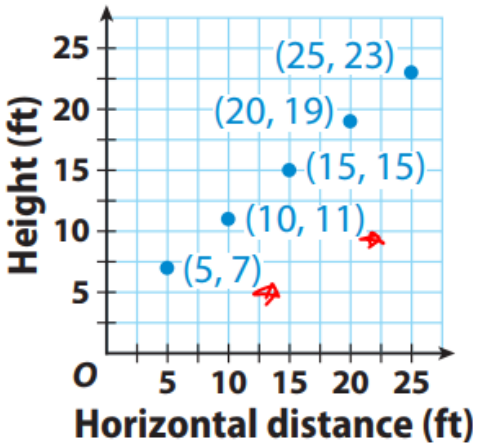
$$y = mx + b$$

m: Slope

b: y-intercept

EXAMPLE 1 Real World

A handrail runs alongside a stairway. As the horizontal distance from the bottom of the stairway changes, the height of the handrail changes. Show that the relationship is linear, and then find the equation for the relationship.



$y = mx + b$

STEP 1 Show that the relationship is linear.

Does it make a line? yes

$\frac{4}{5} \times \frac{5}{1} = \frac{26}{5}$

STEP 2 Write the equation of the linear relationship.

(5,7) (10,11)

Slope

$m = \frac{11-7}{10-5} = \frac{4}{5}$

$m = \frac{4}{5}$

y-int

$7 = \frac{4}{5}(5) + b$

$7 = 4 + b$

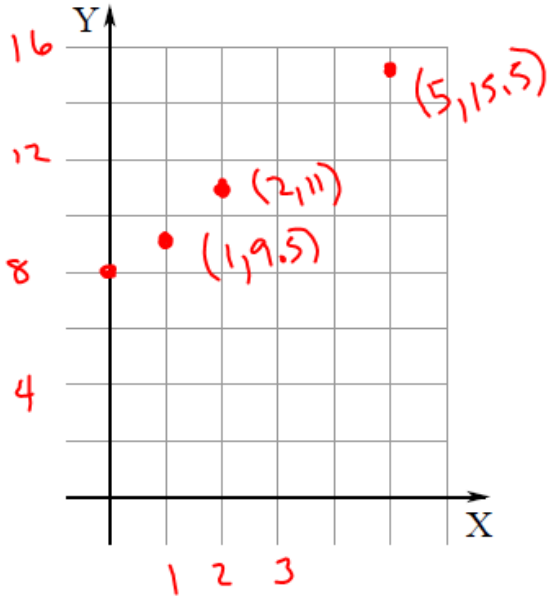
$-4 \quad -4$

$3 = b$

$y = \frac{4}{5}x + 3$

ADDITIONAL EXAMPLE 1

The charge for a cheese pizza changes as the number of toppings changes. Show that the relationship is linear, and then find the equation for the relationship.



Slope

(1, 9.5)

(2, 11)

$m = \frac{11-9.5}{2-1} = \frac{1.5}{1}$

y-int

$9.5 = 1.5(1) + b$

$9.5 = 1.5 + b$

$-1.5 \quad -1.5$

$8 = b$

$y = 1.5x + 8$

YOUR TURN

Find the equation of each linear relationship.

↑ 46 / → 10 = 4/1

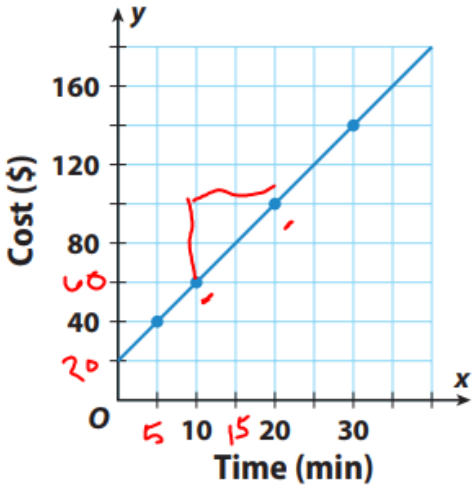
40 = 4(5) + b

46 = 20 + b

−20 −20

20 = b

1.



$y = 4x + 20$

2.

Hours (x)	Number of units (y)
2	480
15	3,600
24	5,760
30	7,200
48	11,520
55	13,200

13 → 3120; 9 → 2160; 6 → 1440; 18 → 4320; 7 → 1680

480 = 240(2) + b

480 = 480 + b

−480 −480

0 = 0 + b

$y = 240x$

$m = \frac{\Delta y}{\Delta x} = \frac{3120}{13} \quad \frac{2160}{9} \quad \frac{1440}{6} \quad \frac{4320}{18} \quad \frac{1680}{7}$

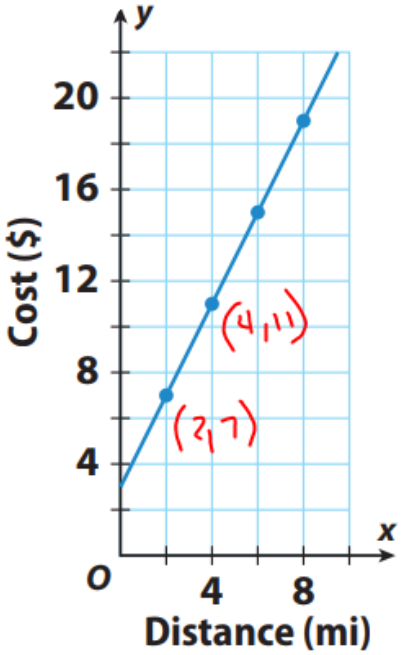
240 240 240 240 240

Making Predictions

EXAMPLE 2

Real World

The graph shows the cost for taxi rides of different distances. Predict the cost of a taxi ride that covers a distance of 6.5 miles.



STEP 1 Write the equation of the linear relationship.

$(2,7)$ $(4,11)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{11-7}{4-2} = \frac{4}{2} = 2$

$7 = 2(2) + b$

$7 = 4 + b$

$-4 \quad -4$

$3 = b$

$y = 2x + 3$

STEP 2

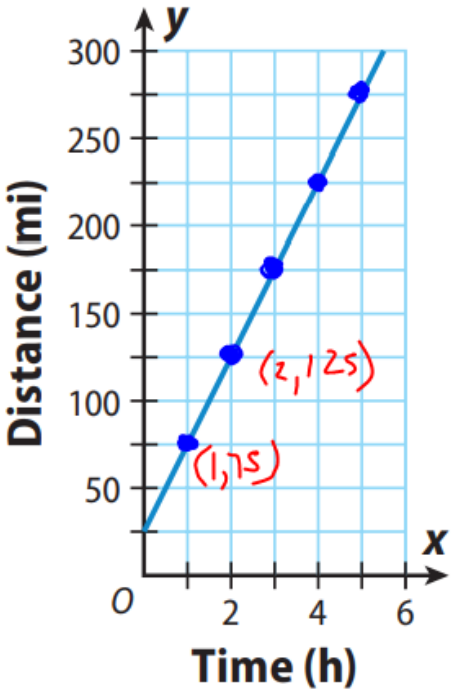
$y = 2(6.5) + 3$

$= 13 + 3$

$y = 16$

ADDITIONAL EXAMPLE 2

The graph shows the distance of a train from a landmark as it travels at a constant speed. Use the graph to predict what the distance will be after 7.5 hours.



Slope

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{125 - 75}{2 - 1} = \frac{50}{1}$

y-int

$y = mx + b$

$75 = 50(1) + b$

$75 = 50 + b$

$-50 \quad -50$

$25 = b$

$y = 50x + 25$

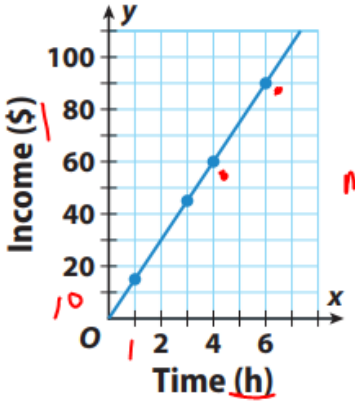
$y = 50(7.5) + 25$

$= 375 + 25$

$y = 400$

YOUR TURN

Paulina's income from a job that pays her a fixed amount per hour is shown in the graph. Use the graph to find the predicted value.



6. Income earned for working 2 hours

$30

7. Income earned for working 3.25 hours

$48.75

8. Total income earned for working for five 8-hour days all at the standard rate $600

(6, 90)
(4, 60)
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$\frac{90 - 60}{6 - 4}$
$\frac{30}{2} = \frac{15}{1}$
$y = mx + b$
$60 = 15(4) + b$
$60 = 60 + b$
$0 = b$
$y = 15(8)$
$= 120$ x's
$y = 15x$
$y = 15(2)$
$y = 15(3.25)$

Vocabulary

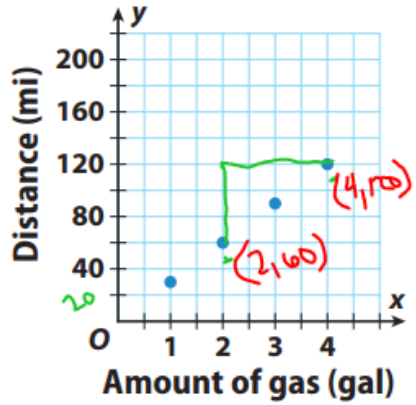
- Bivariate Data - set of data that is made up of two paired variables

- Nonlinear Relationship - Does not make straight line on a graph
  - Variable rate of change

Guided Practice

Use the following graphs to find the equation of the linear relationship. (Example 1)

1.



$\frac{\uparrow 60}{\rightarrow 2} = \frac{60}{2} = \frac{30}{1}$

$\frac{120-60}{4-2} = \frac{60}{2} = \frac{30}{1}$

$y = 30x$

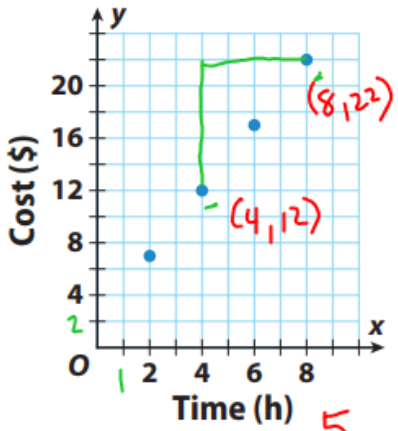
$60 = 30(2) + b$

$60 = 60 + b$

$-60 \quad -60$

$0 = b$

2.



$\frac{\uparrow 10}{\rightarrow 4} = \frac{10}{4} = \frac{5}{2}$

$\frac{22-12}{8-4} = \frac{10}{4} = \frac{5}{2}$

$y = \frac{5}{2}x + 2$

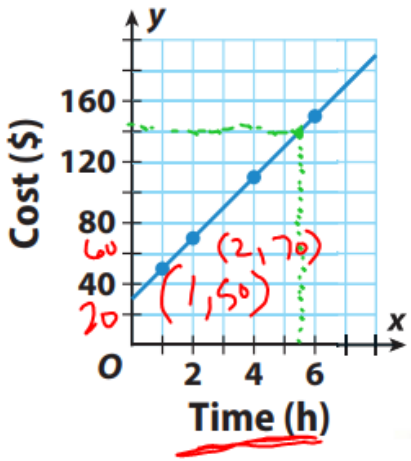
$12 = \frac{5}{2}(4) + b$

$12 = 10 + b$

$-10 \quad -10$

$2 = b$

3. The graph shows the relationship between the number of hours a kayak is rented and the total cost of the rental. Write an equation of the relationship. Then use the equation to predict the cost of a rental that lasts 5.5 hours. (Example 2)



$y = 20x + 30$; $140

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 50}{2 - 1} = \frac{20}{1}$

$50 = 20(1) + b$

$50 = 20 + b$

$-20 \quad -20$

$30 = b$

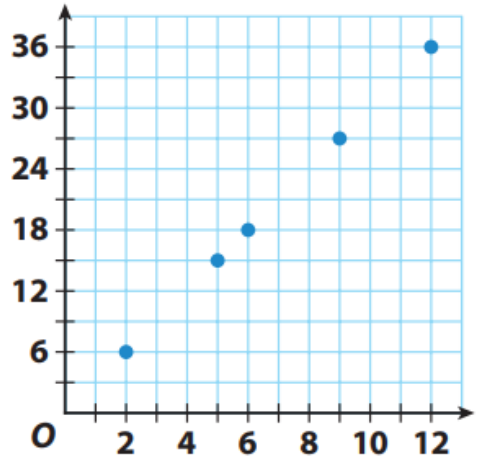
$y = 20(5.5) + 30$

$= 110 + 30$

$= 140$

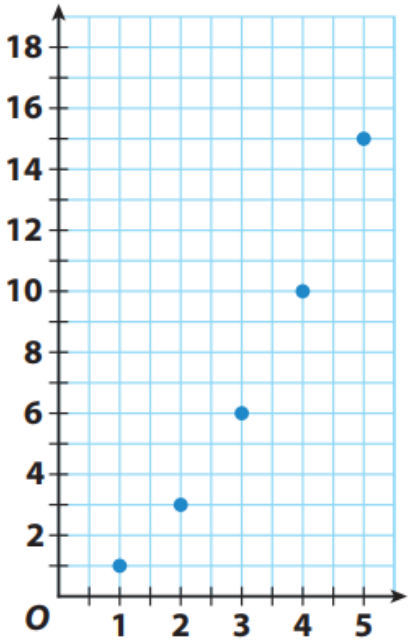
Does each of the following graphs represent a linear relationship? Why or why not? (Explore Activity)

4.



Linear ; constant rate of change

5.



Nonlinear ; variable rate of change