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Solve each equation.

$$1. \quad 5(x+6) + 11 = 25 + 3x$$

$$5x + 30 + 11 = 25 + 3x$$

$$5x + 41 = 25 + 3x$$

$$\begin{array}{r} 8x + 41 = 25 \\ -41 \quad -41 \\ \hline 8x = -16 \\ \hline x = -2 \end{array}$$

$$x = -2$$

LCM(2, 8): 8

$$2. \quad -\frac{3}{8}(-6 - 2y) = \frac{1}{2}(2y - 3) - 1$$

$$-\frac{24}{8} = -3(-6 - 2y) = 4(2y - 3) - 1$$

$$18 + 6y = 8y - 12 - 1$$

$$18 + 6y = 8y - 13$$

$$18 = 2y - 13$$

$$\frac{31}{2} = \frac{2y}{2}$$

$$y = 15.5$$

7.4 Equations with Many Solutions or No Solutions

8.EE.7a

Give examples of linear equations in one variable with one solution, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x=a$, $a=a$, or $a=b$ results (where a and b are different numbers)

When you simplify an equation using the properties of equality, you will find one of three results.

$a =$ any
number

Result	What does this mean?	How many solutions?
$X = a$	When the value of X is a , the equation is a true statement	1
$a = a$	Any value of X makes the equation a true statement	Infinitely Many
$a = b$ where $a \neq b$	There is no value of X that makes the equation a true statement	No Solution

EXAMPLE 1

Use the properties of equality to simplify each equation. Tell whether the final equation is a true statement.

A $4x + 3 = 2x + 13$

$$\begin{array}{r} 4x + 3 = 2x + 13 \\ -4x \quad -4x \\ \hline \end{array}$$

$$\begin{array}{r} -3 = -2x + 13 \\ -13 \quad -13 \\ \hline \end{array}$$

$$\begin{array}{r} -16 = -2x \\ -2 \quad -2 \\ \hline \end{array}$$

$$8 = x$$

True one
Solution

B $4x + 5 = 2(2x + 1) + 3$

$$4x + 5 = 4x + 2 + 3$$

$$\begin{array}{r} 4x + 5 = 4x + 5 \\ -4x \quad -4x \\ \hline \end{array}$$

$$-5 = -5$$

True
Infinitely
Many

C $4x + 2 = 4x + 5$

$$\begin{array}{r} 4x + 2 = 4x + 5 \\ -2 \quad -2 \\ \hline \end{array}$$

$$\begin{array}{r} 4x = 4x + 3 \\ -4x \quad -4x \\ \hline \end{array}$$

$$0 = 3 \text{ False}$$

No Solution

ADDITIONAL EXAMPLE 1

Use the properties of equality to simplify each equation. Tell whether the equation has one, zero, or infinitely many solutions.

A $3x + 6 = 4 + 2x$

$$\begin{array}{r} 3x + 6 = 4 + 2x \\ \underline{-2x} \qquad \underline{-2x} \end{array}$$

$$\begin{array}{r} x + 6 = 4 \\ \underline{-6} \quad \underline{-6} \end{array}$$

$x = -2$ True
One solution

B $3x + 8 = 3(x + 4) + 1$

$$\begin{array}{r} 3x + 8 = 3x + 12 + 1 \\ \underline{-3x} \qquad \underline{-3x} \end{array}$$

$$\begin{array}{r} 8 = 13 \\ \underline{-8} \quad \underline{-8} \end{array}$$

$0 = 5$ False
No Solution

C $3x + 7 = 3(x + 3) + 2$

$$3x + 7 = 3x + 9 + 2$$

$$\begin{array}{r} 3x + 7 = 3x + 11 \\ \underline{-3x} \quad \underline{-3x} \end{array}$$

$$7 = 11$$

False
No Solution

YOUR TURN

Use the properties of equality to simplify each equation. Tell whether the final equation is a true statement.

2. $2x + 1 = 5x + 8$

$$\begin{array}{r} 2x + 1 = 5x + 8 \\ \underline{+8} \quad \underline{-8} \end{array}$$

$$\begin{array}{r} 2x + 9 = 5x \\ \underline{-2x} \quad \underline{-2x} \end{array}$$

$$\frac{9}{3} = \frac{3x}{3}$$

$3 = x$ True
one
solution

3. $3(4x + 3) + 2 = 12x + 7$

$$12x + 9 + 2 = 12x + 7$$

$$\begin{array}{r} 12x + 7 = 12x + 7 \\ \underline{-12x} \quad \underline{-12x} \end{array}$$

$$7 = 7 \text{ True}$$

Infinitely
Many

4. $3x + 9 = 5 + 3x$

$$\begin{array}{r} 3x + 9 = 5 + 3x \\ \underline{-3x} \quad \underline{-3x} \end{array}$$

$$-9 = 5 \text{ False}$$

No Solution

EXAMPLE 2

Write a linear equation in one variable that has no solution.

STEP 1

STEP 2

STEP 3

YOUR TURN

Tell whether each equation has one, zero, or infinitely many solutions.

6. $6 + 3x = x - 8$

7. $8x + 4 = 4(2x + 1)$

YOUR TURN

Complete each equation so that it has the indicated number of solutions.

8. No solution: $3x + 1 = 3x + \underline{\quad}$

9. Infinitely many: $2x - 4 = 2x - \underline{\quad}$

Guided Practice

Use the properties of equality to simplify each equation. Tell whether the final equation is a true statement. (Example 1)

1. $3x - 2 = 25 - 6x$

$$\begin{array}{r} \underline{+6x} \\ \square - 2 = \square \\ \square = \square \\ \underline{\hspace{1cm}} \\ \square x = \square \end{array}$$

$$\begin{array}{r} \square x \\ \square = \square \\ x = \square \end{array}$$

The statement is .

• T/F?
• How many
solutions

$$3x - 2 = 25 - 6x$$

$$\begin{array}{r} 3x - 2 = 25 - 6x \\ \underline{+6x} \qquad \qquad \underline{+6x} \end{array}$$

$$\begin{array}{r} 9x - 2 = 25 \\ \underline{+2} \qquad \underline{+2} \end{array}$$

$$\frac{9x}{9} = \frac{27}{9}$$

$$x = 3$$

True
One
Solution

$$2x - 4 = 2(x - 1) + 3$$

$$2x + -4 = 2(x + -1) + 3$$

$$2x + -4 = 2x + -2 + 3$$

$$\begin{array}{r} 2x + -4 = 2x + 1 \\ \underline{-2x} \qquad \underline{-2x} \end{array}$$

$$-4 = 1$$

False

No Solution